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# Time Dependents Properties Viscoelasticity Models

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**Abstract:** The mechanical properties of many dental materials depend on the rate of loading. They show a behavior intermediate to that of a rigid solid and a viscous liquid. This behavior is called viscoelastic behavior.

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## 1. Introduction

The mechanical properties of many dental materials depend on the rate of loading. They show a behavior intermediate to that of a rigid solid and a viscous liquid. This behavior is called viscoelastic behavior. [1]

Viscoelastic materials are materials showing viscous and elastic behaviors simultaneously. These materials exhibit both properties and a time-dependent strain behavior. *Elastic strain* typically results from stretching but not breakage of atomic or molecular bonds in an ordered solid. The *viscous* component of viscoelastic strain results from the rearrangement of atoms or molecules within amorphous materials. The elastic portion (which stores energy) behaves according to Hooke's law. The viscous portion (which dissipates energy) behaves according to Newton's law. Viscoelastic materials show time-dependent and delayed response when load is applied and removed.[2,3]

Examples of such materials in dentistry are elastomeric and hydrocolloid impression materials, amalgam, waxes, polymers, and orthodontic elastics. Dentin, oral mucosa, and periodontal ligaments also exhibit viscoelastic behavior. [4]

All polymers exhibit viscoelastic properties. The polymer chains exhibit elastic behavior, chains uncoil but they do not slip past one another because of crystalline regions, entanglements, or crosslinks. Thus, they recoil completely when unloaded (i.e. they store the energy used in displacing them). They also exhibit viscous behavior when chains stretch, uncoil, and slip past one another. This produce plastic, irreversible, permanent distortion and partial recovery when unloaded (Figure 1). Friction between the chains causes a net loss of energy after they return to their original positions. [3]

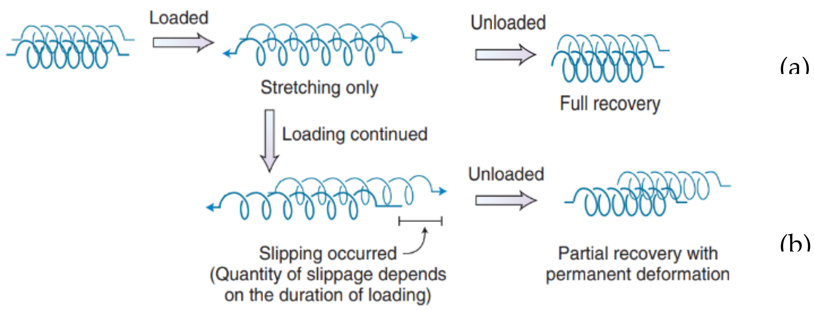



**Figure 1:** (a) Elastic and (b) viscoelastic recovery of polymers.

## 2. The ideal elastic behavior

An ideal elastic material responds instantaneously to applied stress and maintain that strain as long as the load is applied. When this stress is removed, the sample recovers its original dimensions completely and instantaneously (Figure 2). Ideal elastic materials obey ***Hooke's law***, where the stress is directly proportional to the strain independent of the rate at which the body is deformed:

$$\sigma = E\,\varepsilon$$

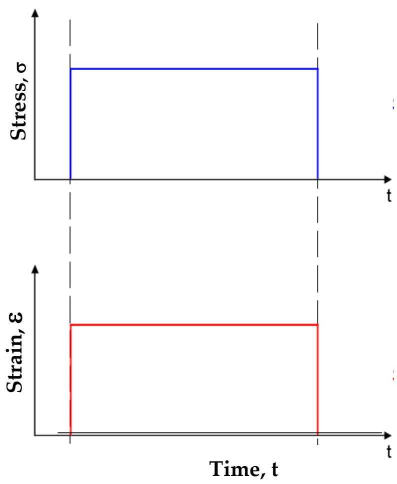



**Figure 2:** Ideally elastic solid response.

Where σ is the stress, E is the modulus of elasticity and ε is the strain.[2]

## 3. The ideal viscous behavior

An ideal viscous material obeys **Newton's law of viscosity**, where the stress is directly proportional to a strain-rate:

$$\dot{\varepsilon} = \frac{\sigma}{\eta}$$

Where $\dot{\varepsilon} = \frac{d\varepsilon}{dt}$ is the strain rate, σ is the stress and η is the viscosity coefficient.

Upon loading, the strain generated is not instantaneous and time dependent. The strain keeps on increasing with time on application of the constant load. When the load is removed, the material does not return to its original dimensions (irreversible deformation) (Figure 3).[2]

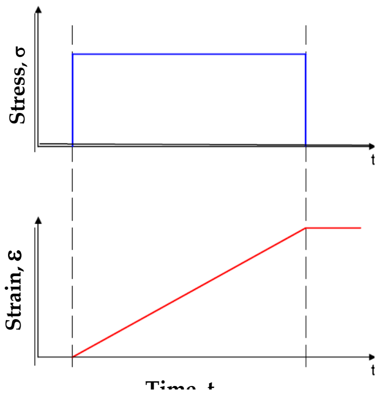



Figure 3: Ideally viscous behavior.

## 4. Anelastic behavior

Anelasticity is a time-dependent elastic behavior. When load is applied, a non-linear (gradual) increase in strain with time. When the load is removed, there is a gradual but complete recovery (Figure 4).[5]

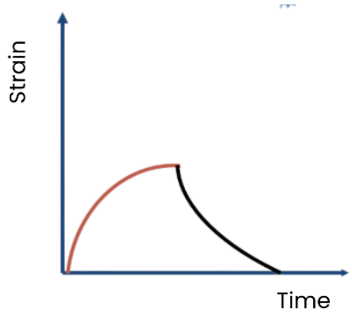



**Figure 4:** Anelastic behavior (delayed elasticity).

## 5. Viscoelastic behavior

This behavior is a combination of ideal elastic, ideal viscous and anelastic. When load is applied, there is an instantaneous elastic strain, followed by a viscous and anelastic time-dependent strain. Upon load removal, there is immediate elastic recovery and gradual anelastic recovery, while the viscous portion will not recover (Figure 5).[5]

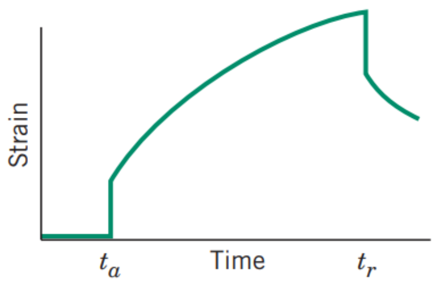



**Figure 5:** Viscoelastic behavior.

## 6. Characteristics of Viscoelastic Materials

There are three main characteristics of viscoelastic materials: *hysteresis, stress relaxation, and creep.*

**a. Hysteresis**

Unlike elastic materials which store all the energy due to deformation, viscoelastic materials dissipate some of this energy by internal frictional mechanisms. As a result, there is a difference between the energy provided to the material and the energy recovered. This makes the loading and unloading curves non-equivalent but form a hysteresis loop (Figure 6) .[6]

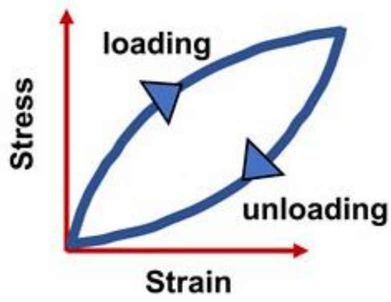



**Figure 6:** Hysteresis.

**b. Stress relaxation**

When a viscoelastic material is strained and this strain is kept constant afterwards, the corresponding stresses induced within the material decrease with time (Figure 7). This is due to a re-arrangement of the material on the molecular or micro-scale. [6]

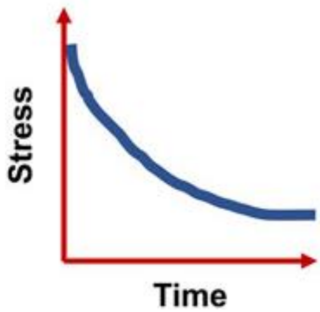



**Figure 7:** Stress relaxation.

**c. Creep**

When a viscoelastic material is loaded at a constant stress, holding that stress for some time, the material will show increased deformation (strain) under constant stress (Figure 8). [6]

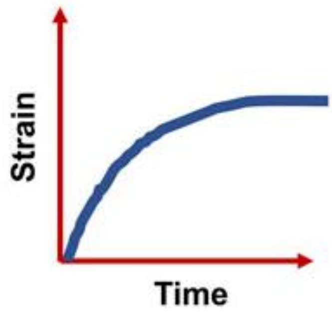



**Figure 8:** Creep.

## 7. Viscoelasticity Models

### The Basic Elements: Spring and Dashpot

**The Spring:**

The spring represents the elastic component (Figure 9), The spring has a spring constant (k). Because of this spring constant, there is a maximum deformation that can be reached (under constant force). [2]

**Figure 9:** Spring model to illustrate ideally elastic behavior.

**F = k X**

Where F is the applied force, k is the spring constant and X is the deformation. So spring reaches an equilibrium situation after it gains maximum deformation.[2]

The spring stores all energy the during deformation. This energy is then available to restore the body to its original shape when these forces are removed.[2]

**The Dashpot:**

It represents the viscous component (Figure 10). The dashpot responds with a strain-rate proportional to stress according to Newton's law of viscosity. When work is done to a dashpot, energy is not stored, instead heat is generated and lost to the environment due to internal friction.[2]

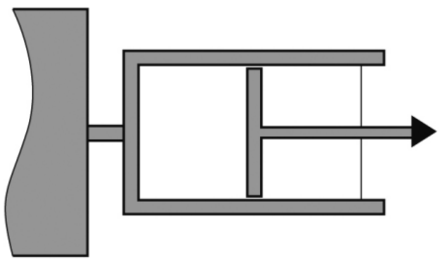


**Figure 10:** The dashpot model to illustrate ideally viscous behavior.

**a. Two-Element Models:**

**i. Maxwell Model**

The Maxwell model can be represented by a purely viscous dashpot and a purely elastic spring connected ***in series*** (Figure 11). Both components can be deflected independently of each other. [2]

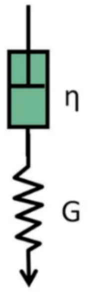



**Figure 11:** Maxwell model.

*Upon loading:* when applying a constant force, only the spring shows immediate deformation until reaching a constant deflection value. Afterwards, the piston of the dashpot begins to move and continue to move as long as the force is applied (Figure 12b).[2]

*Upon load removal:* the spring recoils back elastically (immediately and completely). However, the dashpot remains unchanged. Thus, there is elastic response and a permanent strain but no anelastic recovery (Figure 12b). These kinds of samples remain **partially deformed** due to the viscous portion represented by the Dashpot **(irreversible deformation process)** (Figure 12b).[2]

In Maxwell model, the stress on each element is the same and equal to the imposed stress (called iso-stress model), while the total strain is the sum of the strain in each element.[7]

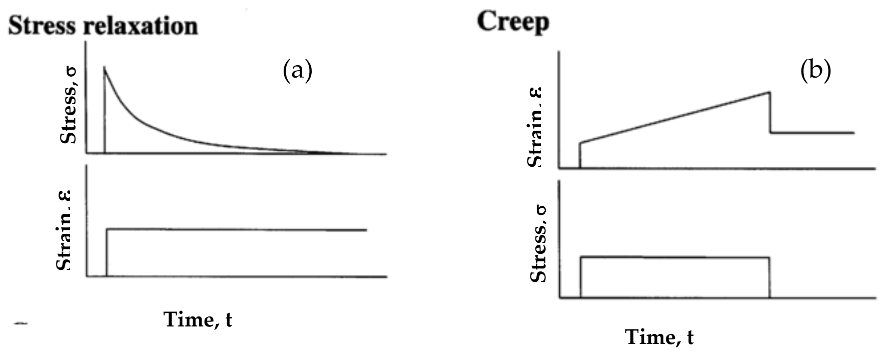



**Figure 12**: (a) Stress relaxation, (b) Creep curves of Maxwell model.

The Maxwell model is **more suitable to explain the stress relaxation of polymer**, as it predicts that stress relax exponentially with time, which is accurate for most polymers (Figure 12a). This model actually represents a **viscoelastic fluid** since it relaxes completely to zero stress and undergoes creep indefinitely. [8]

However, it is **unable to accurately predict creep** as it suggests that strain will increase linearly with time. However, polymers for the most part show the strain rate to be decreasing with time. It is not suitable for modelling materials over long periods of time, as it places no limit upon how much the dashpot can extend. [8]

## ii. Kelvin/Voigt model

The Kelvin/Voigt model can be represented by a purely viscous dashpot and a purely elastic spring connected ***in parallel***. Both components are connected by a rigid frame (Figure 13). (2)

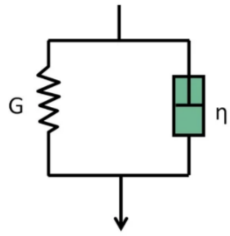



**Figure 13:** Kelvin/Voigt model.

*Upon loading:* when applying a constant force, deformation is increasing continuously. Since the two components are connected by a rigid frame, they can only be deformed together simultaneously and to the same extent. The spring can not undergo immediate deformation because its motion is slowed down by the dashpot. According to this model, the dashpot will only extend to the extension produced in the spring.(2)

*Upon load removal:* the spring has tendency to immediately recoil back to its original shape, and this is the driving force that will cause both components to return to their initial shape. However, this will occur only after a certain period of time due to the presence of the dashpot. These kinds of samples show **delayed but complete recovery (reversible deformation process).** (2)

In Kelvin/Voigt model, the stain on each element is the same (called iso-strain model), while the total stress is the sum of the stresses in each element.(7)

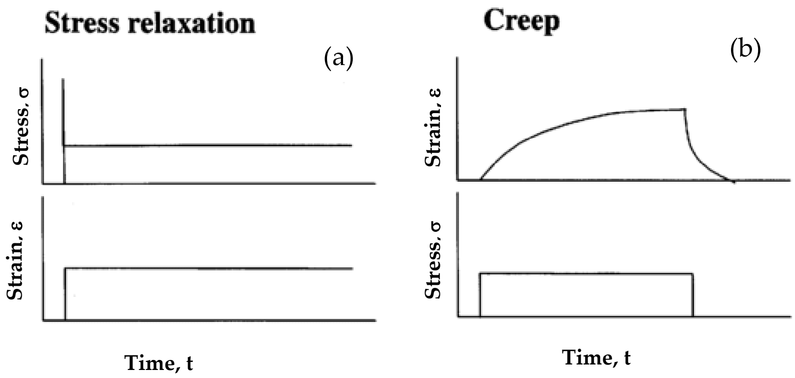



**Figure 14**: (a) Stress relaxation, (b) Creep curves of Kelvin/Voigt model.

This model is suitable for **modelling creep,** because when constant stress is applied, the material deforms at a decreasing rate (It is not linear), i.e. it does not continue to deform as long as stress is applied, rather it reaches an equilibrium deformation (Figure 14b). (8)

However, it is **less accurate in modelling stress**. The stored energy in the spring cannot transfer to the dashpot since the dashpot and spring connect in parallel. So, even though dashpot dissipates its energy, spring cannot dissipate its energy. Thus, the system always has residual stress, and this stress becomes constant after some time (Figure 14a). This model represents a ***viscoelastic solid***.[8]

**b. Three-Element Models:**

The Maxwell and Kelvin models are the simplest viscoelastic models. Maxwell model does not describe creep and Kelvin–Voigt model does not describe stress relaxation accurately. [8]

***The Standard Linear Solid Model (SLS) model*** is a more complex model which combines elements of both the Maxwell and Kelvin-Voigt models. SLS is the simplest model that predicts both phenomena. More realistic material responses can be modelled using more elements.[9]

**i. Zener Models**

They describe a material that will react instantaneously to applied strain and fully recover after a load is removed, because the spring connected in parallel will continue to move the piston of the dashpot back to its original position. Thus, Zener models represent **solids** that undergo a **reversible deformation**.[10]

Zener Model type I (Maxwell representation)

It is obtained by adding a spring in parallel to a Maxwell model (Figure 15a). In this system, the dominant model is the Maxwell model. So, this system is more suitable to explain **stress relaxation** behavior of the solid polymer (Figure 15b).[11]

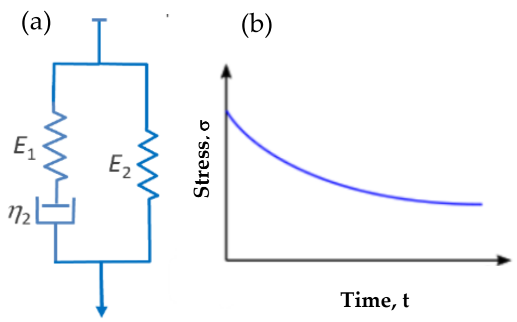



**Figure 15:** SLS Model (a) Zener Model type I, (b) Stress relaxation curve of Zener model.

Zener Model type II (Kelvin–Voigt representation)

It is obtained by adding a spring in series to a Kelvin–Voigt model (Figure 16a). In this system, the dominant model is the Kelvin–Voigt model. So, this system is more suitable to explain **creep** behavior of the solid polymer (Figure 16b).[11]

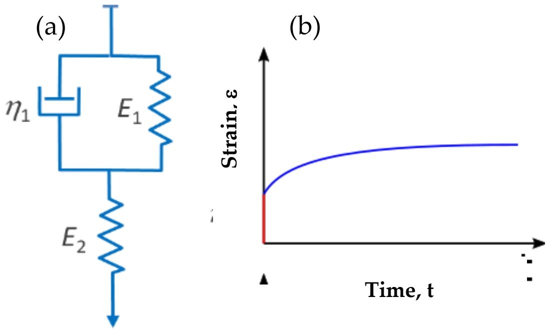



**Figure 16:** SLS Model (a) Zener Model type II, (b) Creep curve of Zener model.

**ii. Jeffreys models (anti-Zener)**

Three-Element Models of Standard Linear Fluid *(SLF Model)*

They describe a material that will only **partially recover** after a load is removed because the piston of the dashpot will not move back to its original position when the load is removed. Thus, anti-Zener models represent a **fluid** that undergoes both a **permanent and elastic deformation**.[10]

Jeffreys Model type I (Kelvin–Voigt representation)

It is obtained by adding a dashpot in series to a Kelvin–Voigt model (Figure 17a). In this system dominant model is the Kelvin–Voigt model. Therefore, this system is more suitable to explain **creep** behavior (Figure 17b). [11]

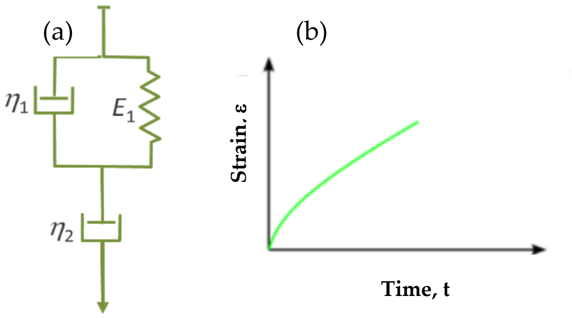



**Figure 17:** SLF Model (a) Jeffreys Model type I (b) Creep curve of Jeffreys model.

Jeffreys Model type II (Maxwell representation)

It is obtained by adding a dashpot in parallel with the Maxwell model (Figure 18a). In this system dominant model is the Maxwell model. Therefore, this system is more suitable to explain **stress relaxation** behavior of molten polymer (Figure 18b). [11]

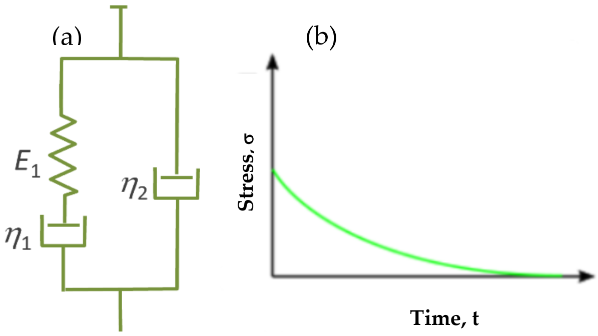



**Figure 18:** SLF Model (a) Jeffreys Model type II (b) Stress relaxation curve Jeffreys model.

**C. Four- Element model (Burgers model)**

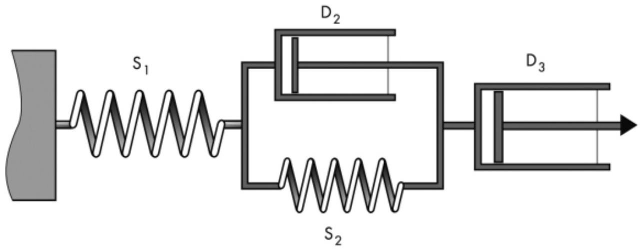



**Figure 19:** Burgers model

It is a combination of the Maxwell model (S1 and D3) and the Kelvin/Voigt model (S2 and D2) joined in series (Figure 19).[2]

*Upon loading:* immediate, step-like deformation of spring $S_1$ .Then delayed deformation of spring S2 and dashpot D2 (like the Kelvin/Voigt model), and continuously increasing deformation of dashpot D3 (D3 can show creep indefinitely).[2]

*Upon load removal:* immediate elastic recovery of spring S1. Delayed recovery of spring S2 and dashpot D2 (like the Kelvin/Voigt model). Dashpot D3 remains completely deflected.[2]

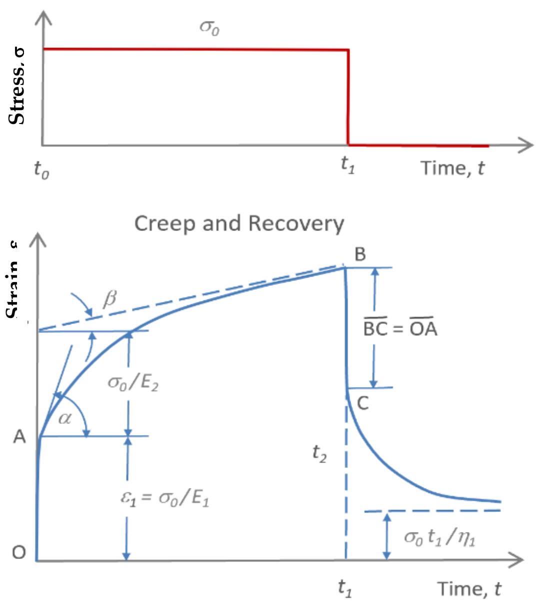



**Figure 20:** Creep and recovery behavior of burgers model.

The four-parameter model provides a crude qualitative representation of the phenomena generally observed with viscoelastic materials: instantaneous elastic strain, retarded elastic strain, viscous flow, instantaneous elastic recovery, retarded elastic recovery, and plastic deformation (permanent set) (Figure 20).[12]

## 8. Testing of Viscoelastic Materials

The viscoelastic behavior of polymeric materials is dependent on both time and temperature; several experimental techniques may be used to measure and quantify this behavior.

### a. The Creep and Recovery Test

This test involves loading a material at *constant stress*, holding that stress for some length of time and then removing the load. There is an instantaneous elastic strain, followed by increasing strain over time known as ***creep strain***. The creep strain normally would increase with an ever-decreasing strain rate, which eventually leads to a constant-strain steady state. The ratio of total strain to an applied constant stress is **creep compliance $J(t)$:**

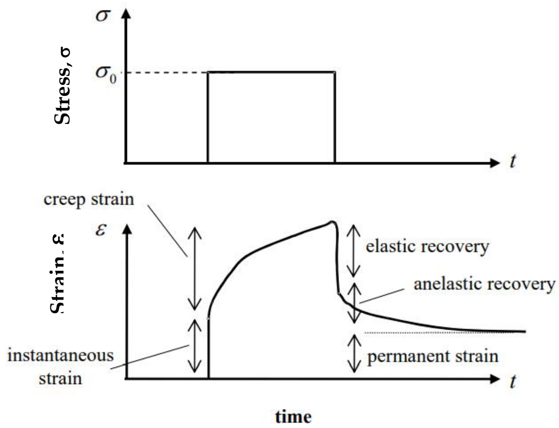
$$J(t) = \frac{\sigma_0}{\varepsilon(t)}$$

where $\sigma_0$ is the constant applied stress and $\varepsilon(t)$ is the time-dependent strain.[7]

In a creep test, the resulting strain for viscoelastic solids increases until it reaches a non-zero equilibrium value, while for viscoelastic fluids the resulting strain increases without bound as time increases.[7]

When unloaded, the elastic strain is recovered immediately. There is then anelastic recovery (strain recovered over time). A permanent strain may then be left in the material (Figure 21).[13]

The creep performance of viscoelastic materials reveals their dimensional stability and their capacity to sustain the load in the long run.[14]




covery test.

*Dental significance of creep*

For a given load at a given time, the low-copper amalgam has a greater strain compared to high-copper amalgam. The greater creep in the low-copper amalgam makes it more susceptible to strain accumulation and fracture, and also marginal breakdown, which can lead to secondary decay.[4]

### b. Stress Relaxation Test

This test involves straining a material at constant strain and then holding that strain. The stress necessary to maintain this strain is measured as a function of time while temperature is held constant. [5]

In a stress relaxation test, viscoelastic solids gradually relax and reach an equilibrium stress greater than zero, while for viscoelastic fluids the stress vanishes to zero (Figure 22).[7]

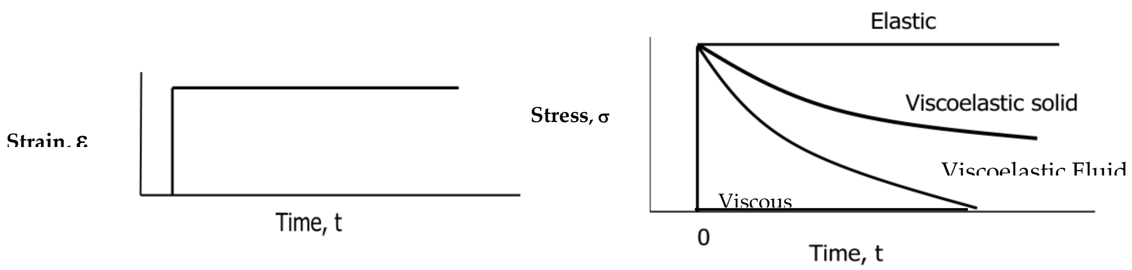



**Figure 22:** Stress relaxation test showing response for an elastic solid, a viscoelastic solid, a viscoelastic liquid and a viscous fluid.

**Relaxation modulus *$E_r(t)$*,** a time-dependent elastic modulus for viscoelastic polymers:

$$E_r(t) = \frac{\sigma(t)}{\varepsilon_0}$$

where $\sigma$(t) is the measured time-dependent stress and $\varepsilon_0$ is the strain level, which is maintained constant. A constant strain $\varepsilon_0$ acts as "input" to the material from time $t_0$, the resulting time-dependent stress is decreasing until a plateau is reached at some later time. [5]

***Dental significance of stress relaxation***

In the evaluation of orthodontic elastic bands. The initial force was much greater with the plastic band, but the decrease in force with time was much less for the latex band. Therefore, plastic bands are useful for applying high forces, although the force decreases rapidly with time, whereas latex bands apply lower forces, but the force decreases slowly with time in the mouth; latex bands are therefore useful for applying more sustained loads.[4]

Similarly, orthodontic aligners that exhibits rapid stress relaxation may express a decreasing amount of orthodontic force once inserted intraorally. The ideal aligner should exhibit a stress relaxation curve that is fairly flat, representing its capability to exert constant and continuous forces over time. Unfortunately, stress relaxation curves for current aligner materials generally follow a pattern of rapid decay within the first 8 hours of application, then diminish to a plateau thereafter. This highlights the importance of measuring forces exerted by aligners not only during the first hours after tray placement inside the oral cavity but also within the first 24- and 48-hours.(15)

## 9. Dental Significance of Viscoelasticity

In viscoelastic materials, the strain rate can change the stress-strain properties.

### Impression materials

For example, the *tear strength of* ***alginate impression material***, is increased about four times when the rate of loading is increased from 2.5 to 25 cm/min. Therefore, alginate impressions should be removed from the mouth quickly to improve its tear resistance. (4)

### Dental amalgam

Another example is the *elastic modulus of* ***dental amalgam***, which is 21 GPa at slow rates of loading and 62 GPa at high rates of loading. Thus, it is particularly important to specify the loading rate with the test results.(4)

### Tissue conditioners

Tissue conditioners also show viscoelastic behavior. An elastic behavior to recover after initial deformation, act as a cushion against the instantaneous cause of pressure, such as biting force. A viscous behavior to allow adaptation to the mucosa, improving the fit of the denture.(16)

### Viscoelastic mucosa.

To get an accurate impression of mucosal tissues in their resting position, patient should not wear the old denture for several hours before taking the impression to allow recovery of the Viscoelastic mucosa. (1)

## 10. Conclusion

By using these mechanical models, we can predict how the material behaves when a load is applied. In general, the more elements a model has, the more accurate it will be in describing the response of real materials. However, the more complex the model, the more material parameters should be evaluated by experiment. The determination of a large number of material parameters might be a difficult, if not an impossible task.

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